

# Lab 5

## Mode Choice (2)

**Yufeng Zhang**  
zhan4879@umn.edu

CEGE-3201: Transportation Engineering

February 21, 2019



# Announcements

- ▶ Report 1 grade statistics:

Mean	Max	Min	Median
3.275(82%)	-	-	3.35

- ▶ Report 2 due date postponed to Mar. 6.

# Objectives

- ▶ Understand Independence of Irrelevant Alternatives (IIA)
- ▶ Build nested logit models
- ▶ Interpret nested logit model

# Independence of Irrelevant Alternatives

- ▶ Mode choice: Blue/red bus paradox
- ▶ Route choice:
  - ▶ Routes =  $\{a+b, a+c, d\}$ ;
  - ▶ Travel time =  $\{T, T, T\}$ ;
  - ▶ Probabilities =  $\{1/3, 1/3, 1/3\}$ ;

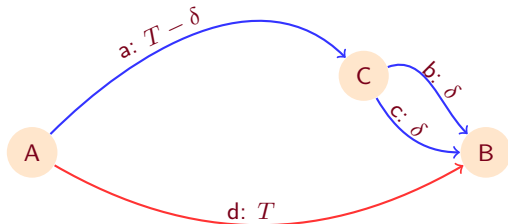


Figure: Route choice problem with overlapping segments

- ▶ In reality...

# Decision Making Processes

Multinomial logit model (MNL) and nested logit model (NL).

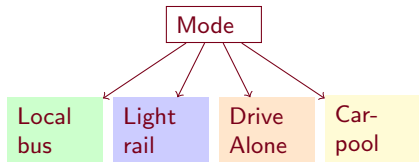


Figure: Single level decision making process

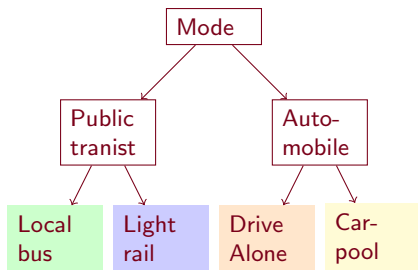
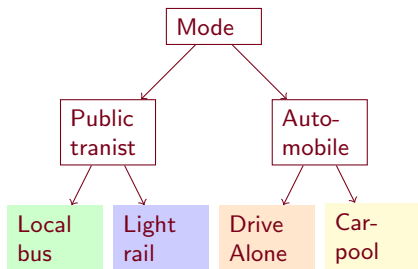


Figure: Nested decision making process

# Conditional Probabilities



First, choose between public transit and automobile;  
 Then, choose between either local bus and light rail or drive alone and car pool.

Figure: Nested decision making process

$$P(\text{local bus}) = P(\text{local bus}|\text{public transit})P(\text{public transit})$$

$$P(\text{drive alone}) = P(\text{drive alone}|\text{automobile})P(\text{automobile})$$

# Calculate Conditional Probabilities

There are various ways to estimate parameters of nested logit models.

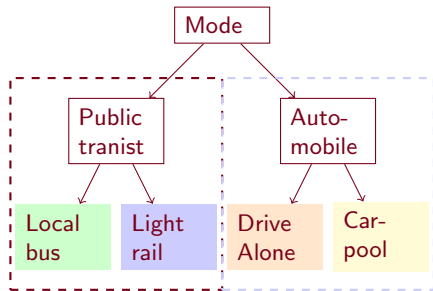


Figure: Conditional probability

$$P(LB|PT) = \frac{\exp \frac{V_{LB}}{u_{PT}}}{\exp \frac{V_{LB}}{u_{PT}} + \exp \frac{V_{LR}}{u_{PT}}}$$

$$P(LR|PT) = \frac{\exp \frac{V_{LR}}{u_{PT}}}{\exp \frac{V_{LB}}{u_{PT}} + \exp \frac{V_{LR}}{u_{PT}}}$$

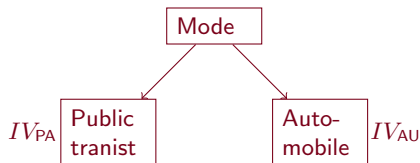
$$P(DA|AU) = \frac{\exp \frac{V_{DA}}{u_{AU}}}{\exp \frac{V_{DA}}{u_{AU}} + \exp \frac{V_{CP}}{u_{AU}}}$$

$$P(CP|AU) = \frac{\exp \frac{V_{CP}}{u_{AU}}}{\exp \frac{V_{DA}}{u_{AU}} + \exp \frac{V_{CP}}{u_{AU}}}$$

- ▶  $u_{PT}$  and  $u_{AU}$  are scale parameters that are specific to public transit and automobile branches respectively. These parameters will be also estimated from data.

# Calculate Branch Probabilities

Utilities of elementary alternatives enter branch utilities as inclusive values (IV).



$$IV_{PT} = \ln\left(\exp \frac{V_{LR}}{u_{PT}} + \exp \frac{V_{LB}}{u_{PT}}\right)$$

$$IV_{AU} = \ln\left(\exp \frac{V_{DA}}{u_{AU}} + \exp \frac{V_{CP}}{u_{AU}}\right)$$

Figure: Branch choice

Calculate probabilities of choosing between public transit and automobile.

$$P(PT) = \frac{\exp(u_{PT}IV_{PT})}{\exp(u_{PT}IV_{PT}) + \exp(u_{AU}IV_{AU})}$$

$$P(AU) = \frac{\exp(u_{AU}IV_{AU})}{\exp(u_{PT}IV_{PT}) + \exp(u_{AU}IV_{AU})}$$



# Calculate Alternative Probabilities

Eg.

$$P(\text{CP}) = P(\text{CP}|\text{AU})P(\text{AU})$$

# Build Nested Logit Model Using R

R packages:

- ▶ R package — “mlogit”
- ▶ Data set “TravelMode” from R package — “AER”
  - ▶ How many choice situations are there in data sets?
  - ▶ How many alternatives are there in data sets?
  - ▶ What attributes are recorded in data sets?
- ▶ Follow the steps of constructing new data set in file “Nested Logit Models.pdf”